# BPS surface observables in six-dimensional $(2,0)$ theory 

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AbSTRACT: The supergroup $O S p\left(8^{*} \mid 4\right)$, which is the superconformal group of $(2,0)$ theory in six dimensions, is broken to the subgroup $O S p(4 \mid 2) \times O S p(4 \mid 2)$ by demanding the invariance of a certain product in a superspace with eight bosonic and four fermionic dimensions. We show that this is consistent with the symmetry breaking induced by the presence of a flat two-dimensional BPS surface in the usual $(2,0)$ superspace, which has six bosonic and sixteen fermionic dimensions.

Keywords: Superspaces, Space-Time Symmetries, Field Theories in Higher Dimensions. Supersymmetry Breaking.

## Contents

1. Introduction 1
2. The superconformal group 2
3. Breaking the superconformal symmetry 3
4. The superconformal space 6

## 1. Introduction

One of the main motivations for the study of superconformal $(2,0)$ theories in six dimensions is their relation to maximally supersymmetric $\mathcal{N}=4$ Yang-Mills theory in four dimensions [1-5]. This viewpoint provides an important insight into the $S$-duality of the four-dimensional theory [6-8], since the electrically and magnetically charged particles are interpreted as different windings of self-dual strings on the compactification torus. $S$-duality then becomes a simple consequence of the $\operatorname{SL}(2, \mathbb{Z})$ modular invariance.

Another intriguing connection between these theories is the following: In the fourdimensional $\mathcal{N}=4$ Yang-Mills theory, there are two important observables that have attracted much interest recently. These are the Wilson operator and its dual, the 't Hooft operator, which both are associated with closed spatial curves. From the six-dimensional perspective, these should correspond to different windings of a single type of BPS surface observable in $(2,0)$ theory [9-12]. The connection between the Wilson and the 't Hooft operators is also indicated by the results in refs. [13, (14].

The study of BPS Wilson loops in $\mathcal{N}=4$ supersymmetric Yang-Mills theory is also motivated from the AdS/CFT correspondence 15-17. This viewpoint provides important information about string theory on $A d S_{5} \times S^{5}$. Similarly, the study of BPS surface observables in $(2,0)$ theory might be related to quantities in $M$-theory on $A d S_{7} \times S^{4}$.

The purpose of the present paper is to investigate in what way the superconformal group $\operatorname{OSp}\left(8^{*} \mid 4\right)$ is broken if a spatial two-dimensional flat surface is introduced in the $(2,0)$ superspace. This is inspired by the work in ref. 18], where it is shown that a line in four-dimensional $\mathcal{N}=4$ Yang-Mills theory breaks the superconformal group $\operatorname{PSU}(2,2 \mid 4)$ to the subgroup $\operatorname{OSp}(2,2 \mid 4)$. We will use the results obtained in refs. [19, 20] to simplify the treatment of the superconformal transformations, and we expect that the results obtained here will be useful in future studies of surface operators in $(2,0)$ theory.

The outline of this paper is as follows: section 2 reviews the superconformal group and the transformations therein, focusing on the possibility of a linear formulation in a superspace with eight bosonic and four fermionic dimensions. In section 3, we introduce a
way of breaking the superconformal symmetry of the theory and show that the remaining unbroken symmetry leaves a two-dimensional surface invariant. Finally, section contains an attempt to describe this surface from an eight-dimensional perspective.

## 2. The superconformal group

Consider the supergroup $O S p\left(8^{*} \mid 4\right)$, which is the superconformal group relevant for $(2,0)$ theory in six dimensions [21, 22]. By definition, this group leaves the inner product

$$
\begin{equation*}
y \cdot z \equiv I^{A B} y_{A} z_{B} \tag{2.1}
\end{equation*}
$$

invariant. In this relation, $y_{A}$ and $z_{B}$ denote coordinate vectors in a superspace with eight bosonic and four fermionic dimensions, while the graded symmetric tensor $I^{A B}$ is the corresponding (inverse) metric. The superindices and the metric will be further explained below.

The generator of superconformal transformations is denoted by $J_{A B}$, which is graded antisymmetric and obeys the (anti)commutation relations

$$
\begin{equation*}
\left[J_{A B}, J_{C D}\right\}=-\frac{1}{2}\left(I_{B C} J_{A D}-(-1)^{A B} I_{A C} J_{B D}-(-1)^{C D} I_{B D} J_{A C}+(-1)^{A B+C D} I_{A D} J_{B C}\right) \tag{2.2}
\end{equation*}
$$

where the bracket in the left hand side is an anticommutator if both its entries are fermionic, otherwise it is a commutator. A factor $(-1)^{A}$ is positive if $A$ is a bosonic index, and negative if it is fermionic.

The corresponding coordinate transformation is given by

$$
\begin{equation*}
\delta y_{A}=-\pi^{C D} y_{C} I_{D A} \tag{2.3}
\end{equation*}
$$

where the graded antisymmetric quantity $\pi^{C D}$ contains the infinitesimal parameters. It is easily verified that a transformation of this form indeed leaves the inner product in eq. (2.1) invariant.

It is illustrative to decompose the quantities introduced above into more familiar ones [23, 19, 20]. The coordinate vector may be written as $y_{A}=\left(y_{\hat{\alpha}}, y^{a}\right)=\left(y_{\alpha}, y^{\alpha}, y^{a}\right)$. In this expression, the bosonic index $\hat{\alpha}=(1, \ldots, 8)$ is a chiral $\operatorname{SO}(6,2)$ spinor index, which may be further decomposed into one chiral $\mathrm{SO}(5,1)$ spinor index $\alpha=(1, \ldots, 4)$ (subscript) and one anti-chiral $\mathrm{SO}(5,1)$ spinor index $\alpha=(1, \ldots, 4)$ (superscript). Finally, the fermionic index $a=(1, \ldots, 4)$ is an $\mathrm{SO}(5)$ spinor index.

In this notation, the superconformal generators become 24, 23]

$$
J_{A B}=\left(\begin{array}{ccc}
\frac{1}{2}{ }_{\alpha \beta} & \frac{1}{2} M_{\alpha}{ }^{\beta}+\frac{1}{4} \delta_{\alpha}{ }^{\beta} D & \frac{i}{2 \sqrt{2}} Q_{\alpha}^{b}  \tag{2.4}\\
-\frac{1}{2} M_{\beta}{ }^{\alpha}-\frac{1}{4} \delta_{\beta}{ }^{\alpha} D & -\frac{1}{2} K^{\alpha \beta} & \frac{i}{2 \sqrt{2}} \Omega^{b c} S_{c}^{\alpha} \\
-\frac{i}{2 \sqrt{2}} Q_{\beta}^{a} & -\frac{i}{2 \sqrt{2}} \Omega^{a c} S_{c}^{\beta} & i U^{a b}
\end{array}\right)
$$

while the superspace metric is written as

$$
I_{A B}=\left(\begin{array}{ccc}
0 & \delta_{\alpha}{ }^{\beta} & 0  \tag{2.5}\\
\delta^{\alpha} & 0 & 0 \\
0 & 0 & i \Omega^{a b}
\end{array}\right)
$$

Together with eq. (2.2), these definitions reproduce all commutation relations of the sixdimensional superconformal algebra with conventions as in ref. [23]. We have also introduced the $\mathrm{SO}(5)$ invariant antisymmetric tensor $\Omega^{a b}$ in the purely fermionic piece of the superspace metric.

In the inner product defined in eq. (2.1), the inverse superspace metric appears, i.e., the metric with superscript indices. This is written as

$$
I^{A B}=\left(\begin{array}{ccc}
0 & \delta^{\alpha}{ }_{\beta} & 0  \tag{2.6}\\
\delta_{\alpha}{ }^{\beta} & 0 & 0 \\
0 & 0 & -i \Omega_{a b}
\end{array}\right)
$$

in this basis, which makes the relation

$$
\begin{equation*}
I_{A B} I^{B C}=\delta_{A}^{C} \tag{2.7}
\end{equation*}
$$

valid (which is essential if we want to raise and lower indices). This requires that $\Omega_{a b} \Omega^{b c}=$ $\delta_{a}{ }^{c}$.

## 3. Breaking the superconformal symmetry

Consider the subgroup $H \subset O S p\left(8^{*} \mid 4\right)$ which leaves the product

$$
\begin{equation*}
y \circ z \equiv \hat{I}^{A B} y_{A} z_{B} \tag{3.1}
\end{equation*}
$$

invariant. In this expression, we demand the matrix $\hat{I}^{A B}$ to satisfy

$$
\begin{align*}
I_{A B} \hat{I}^{A B} & =0 \\
I_{B C} \hat{I}^{A B} \hat{I}^{C D} & =I^{A D} . \tag{3.2}
\end{align*}
$$

Explicitly, we choose our basis such that

$$
\hat{I}^{A B}=\left(\begin{array}{cccccc}
0 & 0 & \delta^{\dot{\alpha}}{ }_{\dot{\beta}} & 0 & 0 & 0  \tag{3.3}\\
0 & 0 & 0 & -\delta^{\dot{\alpha}} & { }_{\beta} & 0 \\
0 \\
\delta_{\dot{\alpha}} \dot{\beta} & 0 & 0 & 0 & 0 & 0 \\
0 & -\delta_{\dot{\alpha}}^{\dot{\beta}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \epsilon_{\dot{a} \dot{b}} & 0 \\
0 & 0 & 0 & 0 & 0 & -\epsilon_{\dot{a} \dot{b}}
\end{array}\right),
$$

where all indices in the matrix are fundamental $\operatorname{SU}(2)$ indices, taking the values 1 and 2 . In this basis, the $\mathrm{SO}(5)$ invariant tensor is written as

$$
\Omega_{a b}=\left(\begin{array}{cc}
i \epsilon_{\dot{a} \dot{b}} & 0  \tag{3.4}\\
0 & i \epsilon_{\dot{a} \dot{b}}
\end{array}\right)
$$

where the antisymmetric $\operatorname{SU}(2)$ invariant tensor $\epsilon_{\dot{a} \dot{b}}$ is defined such that $\epsilon_{12}=1$. This means that we have decomposed the $\mathrm{SO}(5,1)$ spinor indices according to $\alpha=(\dot{\alpha}, \dot{\alpha})$, but
also the $\mathrm{SO}(5)$ spinor index as $a=(\dot{a}, a \dot{a})$. In total, this leaves us with six different $\mathrm{SU}(2)$ indices. Note that the indices denoted by Greek letters, originating from the bosonic piece of the superindex, may not be raised or lowered; superscript and subscript indices indicate different representations.

Comparing the expression for $\hat{I}^{A B}$ in eq. (3.3) with the inverse superspace metric $I^{A B}$ in eq. (2.6), we see that they differ only by some signs. The inspiration for this may be found in ref. [25], where the $R$-symmetry group $\mathrm{SO}(5)$ is spontaneously broken to $\mathrm{SO}(4) \simeq \mathrm{SU}(2) \times \mathrm{SU}(2)$ by selecting a specific unit vector $\hat{\Phi}^{a b}$ as

$$
\hat{\Phi}^{a b}=\left(\begin{array}{cc}
i \epsilon^{\dot{a} \dot{b}} & 0  \tag{3.5}\\
0 & -i \epsilon^{a \dot{a}}
\end{array}\right) .
$$

The quantity in eq. (3.3) generalizes this idea to the entire superconformal space. Specifically, we note that eq. (3.2) is analogous to the relation $\Omega_{a b} \hat{\Phi}^{a b}=0$ from ref. [25], which makes $\hat{\Phi}^{a b}$ an $\mathrm{SO}(5)$ vector.

To investigate the properties of the subgroup $H$, let us see which $\operatorname{OSp}\left(8^{*} \mid 4\right)$ generators leave the product in eq. (3.1) invariant. By applying the transformation in eq. (2.3), we find that the product is invariant if the generator $J_{A B}$ is of the form

$$
\hat{J}_{A B}=\left(\begin{array}{cccccc}
J_{\dot{\alpha} \dot{\beta}} & 0 & J_{\dot{\alpha}}{ }^{\dot{\beta}} & 0 & J_{\dot{\alpha}}{ }^{\dot{b}} & 0  \tag{3.6}\\
0 & J_{\dot{\alpha} \dot{\beta}} & 0 & J_{\dot{\alpha}}^{\dot{\beta}} & 0 & J_{\dot{\alpha}}{ }^{\dot{b}} \\
J^{\dot{\alpha}} & 0 & J^{\dot{\alpha} \dot{\beta}} & 0 & J^{\dot{\alpha} \dot{b}} & 0 \\
0 & J^{\dot{\alpha}} & 0 & J^{\dot{\alpha} \dot{\beta}} & 0 & J^{\alpha \dot{\alpha} \dot{b}} \\
J^{\dot{a}}{ }_{\dot{\beta}} & 0 & J^{\dot{\alpha} \dot{\beta}} & 0 & J^{\dot{a} \dot{b}} & 0 \\
0 & J^{\dot{a}} & & 0 & J^{\dot{\alpha} \dot{\beta}} & 0
\end{array} J^{\dot{a} \dot{b}}\right) .
$$

By rearranging the indices, this matrix is easily brought to a block-diagonal form, indicating that the subgroup $H$ is a product of two identical groups; one associated with dotted and one with primed indices. These groups each have nine bosonic and eight fermionic generators, to be compared with the 38 bosonic and 32 fermionic generators of $\operatorname{OSp}\left(8^{*} \mid 4\right)$. Thus, half the supersymmetries and half the special supersymmetries are left unbroken.

Next, consider how $H$ acts on the $(2,0)$ superspace with six bosonic coordinates, denoted by $x^{\alpha \beta}=-x^{\beta \alpha}$, and sixteen fermionic coordinates, $\theta_{a}^{\alpha}$. The coordinate transformations induced by the full $\operatorname{OSp}\left(8^{*} \mid 4\right)$ supergroup are given by [23, 26]

$$
\begin{align*}
\delta x^{\alpha \beta}= & a^{\alpha \beta}-\omega_{\gamma}^{[\alpha} x^{\beta] \gamma}+\lambda x^{\alpha \beta}+4 c_{\gamma \delta} x^{\gamma \alpha} x^{\beta \delta}-i \Omega^{a b} \eta_{a}^{[\alpha} \theta_{b}^{\beta]}- \\
& -c_{\gamma \delta} \theta^{\gamma} \cdot \theta^{[\alpha} \theta^{\beta]} \cdot \theta^{\delta}-i \rho_{\gamma}^{c} \theta_{c}^{[\alpha}\left(2 x^{\beta] \gamma}-i \theta^{\beta]} \cdot \theta^{\gamma}\right)  \tag{3.7}\\
\delta \theta_{a}^{\alpha}= & \left(\omega_{\gamma}{ }^{\alpha}-4 c_{\gamma \delta} x^{\alpha \delta}-2 i c_{\gamma \delta} \theta^{\alpha} \cdot \theta^{\delta}+2 i \rho_{\gamma}^{c} \theta_{c}^{\alpha}\right) \theta_{a}^{\gamma}+\frac{1}{2} \lambda \theta_{a}^{\alpha}+  \tag{3.8}\\
& +\eta_{a}^{\alpha}-\Omega_{a c} \rho_{\gamma}^{c}\left(2 x^{\gamma \alpha}-i \theta^{\gamma} \cdot \theta^{\alpha}\right)+v_{a c} \Omega^{c d} \theta_{d}^{\alpha},
\end{align*}
$$

where the conventions for the various parameters may be found in ref. [23]. If we split the indices on the coordinates and restrict ourselves to the transformations contained in $H$, we
find that a configuration where

$$
\begin{align*}
x^{\dot{\alpha} \dot{\beta}} & =0 \\
\theta_{\dot{b}}^{\dot{\alpha}} & =0  \tag{3.9}\\
\theta_{\dot{b}}^{\dot{\alpha}} & =0
\end{align*}
$$

is left invariant. This corresponds naturally to a flat two-dimensional surface embedded in the six-dimensional space-time, which in addition breaks half the supersymmetry of the theory and also breaks the $R$-symmetry group. The remaining bosonic symmetry transformations, contained in the supergroup $H$, are interpreted as conformal transformations on the surface and rotations in the transverse space, together with the $R$-symmetry rotations in the space transverse to $\hat{\Phi}^{a b}$.

It is interesting to note that the surface specified in eq. (3.9) is necessarily associated to a direction in $R$-symmetry space. This means that the presence of the surface automatically breaks the $R$-symmetry group from $\mathrm{SO}(5)$ to $\mathrm{SO}(4)$. This phenomenon is a consequence of the breaking of supersymmetry.

Let us investigate the properties of the supergroup $H$ : The bosonic subgroup mentioned above is $\mathrm{SO}(3,1) \times \mathrm{SO}(3,1) \times \mathrm{SO}(4)$, provided that the surface is spatial. This means quite naturally that the bosonic subgroup of each of the two factors in $H$ is $\operatorname{SO}(3,1) \times \operatorname{SU}(2)$. The most plausible supergroup with this bosonic subgroup is $\operatorname{OSp}(3,1 \mid 2)$; the next step is to fit the generators contained in $\hat{J}_{A B}$ into this structure.

Leaving questions concerning signature aside, the supergroup $\operatorname{OSp}(4 \mid 2)$ is defined through the commutation relations

$$
\begin{align*}
& {\left[A_{\dot{\alpha} \dot{\beta}}, R_{\dot{\gamma}, \dot{\mu}}^{\dot{a}}\right]=\epsilon_{\dot{\gamma}(\dot{\alpha}} R_{\dot{\beta}), \dot{\mu}}^{\dot{a}} \quad\left[A_{\dot{\alpha} \dot{\beta}}, A_{\dot{\gamma} \dot{\delta}}\right]=\epsilon_{\dot{\gamma}(\dot{\alpha}} A_{\dot{\beta}) \dot{\delta}}+\epsilon_{\dot{\delta}(\dot{\alpha}} A_{\dot{\beta}) \dot{\gamma}}} \\
& {\left[B_{\dot{\mu} \dot{\nu}}, R_{\dot{\alpha}, \dot{\rho}}^{\dot{\dot{p}}}\right]=\epsilon_{\dot{\rho}(\dot{\mu}} R_{\dot{\alpha}, \dot{\nu})}^{\dot{a}} \quad\left[B_{\dot{\mu} \dot{\nu}}, B_{\dot{\rho} \dot{\sigma}}\right]=\epsilon_{\dot{\rho}(\dot{\mu}} B_{\dot{\nu}) \dot{\sigma}}+\epsilon_{\dot{\sigma}(\dot{\mu}} B_{\dot{\nu}) \dot{\rho}}}  \tag{3.10}\\
& {\left[C^{\dot{a} \dot{b}}, R_{\dot{\alpha}, \dot{\mu}}^{\dot{\dot{u}}}\right]=\epsilon^{\dot{c}(\dot{a}} R_{\dot{\alpha}, \dot{\mu}}^{\dot{b})} \quad\left[C^{\dot{a} \dot{b}}, C^{\dot{c} \dot{d}}\right]=\epsilon^{\dot{c}(\dot{a}} C^{\dot{b}) \dot{d}}+\epsilon^{\dot{d}(\dot{a}} C^{\dot{b}) \dot{c}},}
\end{align*}
$$

and the anti-commutation relation

$$
\begin{equation*}
\left\{R_{\dot{\alpha}, \dot{\mu}}^{\dot{a}}, R_{\dot{\beta}, \dot{\nu}}^{\dot{b}}\right\}=\epsilon^{\dot{a} \dot{b}} \epsilon_{\dot{\mu} \dot{\nu}} A_{\dot{\alpha} \dot{\beta}}+\epsilon^{\dot{a} \dot{b}} \epsilon_{\dot{\alpha} \dot{\beta}} B_{\dot{\mu} \dot{\nu}}-2 \epsilon_{\dot{\alpha} \dot{\beta}} \epsilon_{\dot{\mu} \dot{\nu}} C^{\dot{a} \dot{b}} \tag{3.11}
\end{equation*}
$$

where we note three three-component bosonic generators (denoted by $A_{\dot{\alpha} \dot{\beta}}, B_{\dot{\mu} \dot{\nu}}$ and $C^{\dot{a} \dot{b}}$, all symmetric) and one eight-component fermionic generator $R_{\dot{\alpha}, \dot{\mu}}^{\dot{\mu}}$. The bosonic generators appear with different weights in the right-hand side of the anticommutator in eq. (3.11). Similar relations hold for the version with primed indices.

Let us see if we may fit these generators into the structure for $\hat{J}_{A B}$ in eq. (3.6). This procedure ultimately fixes the weights in the right-hand side of eq. (3.11) to these particular values. Define

$$
\begin{align*}
J_{\dot{\alpha}}^{\dot{a}} & =\frac{1}{2}\left(R_{\dot{\alpha}, 1}^{\dot{a}}+R_{\dot{\alpha}, 2}^{\dot{a}}\right)  \tag{3.12}\\
J^{\dot{\alpha} \dot{a}} & =-\frac{1}{4} \epsilon^{\dot{\alpha} \dot{\gamma}}\left(R_{\dot{\gamma}, 1}^{\dot{a}}-R_{\dot{\gamma}, 2}^{\dot{a}}\right),
\end{align*}
$$

where we have let the $\dot{\mu}$-index take specific values. By requiring these generators to satisfy eq. (2.2), we find that we need to take

$$
\begin{align*}
J_{\dot{\alpha} \dot{\beta}} & =-\frac{1}{2} \epsilon_{\dot{\alpha} \dot{\beta}}\left(B_{11}+2 B_{12}+B_{22}\right) \\
J^{\dot{\alpha} \dot{\beta}} & =-\frac{1}{6} \epsilon^{\dot{\alpha} \dot{\beta}}\left(B_{11}-2 B_{12}+B_{22}\right)  \tag{3.13}\\
J_{\dot{\alpha}}^{\dot{\beta}} & =-\frac{1}{2} \epsilon^{\dot{\beta} \dot{\gamma}} A_{\dot{\alpha} \dot{\gamma}}+\frac{1}{4} \delta_{\dot{\alpha}}^{\dot{\beta}}\left(B_{11}-B_{22}\right) \\
J^{\dot{a} \dot{b}} & =-C^{\dot{a} \dot{b}} .
\end{align*}
$$

In this way, we have constructed an explicit isomorphism between the unbroken subgroup $H \subset O S p\left(8^{*} \mid 4\right)$ and the supergroup $\operatorname{OSp}(4 \mid 2) \times \operatorname{OSp}(4 \mid 2)$.

So far, we have considered a flat infinitely extended two-dimensional surface. This is in fact a special case; in general we should consider closed surfaces, such as two-spheres. The moduli space of possible such surfaces is parametrized by the coset $\operatorname{OSp}\left(8^{*} \mid 4\right) /[\operatorname{OSp}(4 \mid 2) \times$ $\operatorname{OSp}(4 \mid 2)$ ], and has 20 bosonic and 16 fermionic dimensions.

## 4. The superconformal space

The purpose of this section is to investigate if there is another way, based on an eightdimensional formulation, of showing that the product in eq. (3.1) is associated with a surface in six dimensions.

In ref. [19], the connection between the superconformal space (with eight bosonic dimensions) and the ( 2,0 ) superspace (with six bosonic dimensions) was accomplished by requiring the fields to live on a projective supercone, defined by

$$
\begin{equation*}
I^{A B} y_{A} y_{B}=0 . \tag{4.1}
\end{equation*}
$$

An explicit solution to this condition is given by

$$
\begin{align*}
y^{a} & =\sqrt{2} \Omega^{a b} \theta_{b}^{\beta} y_{\beta} \\
y^{\alpha} & =\left(2 x^{\alpha \beta}-i \Omega^{a b} \theta_{a}^{\alpha} \theta_{b}^{\beta}\right) y_{\beta} . \tag{4.2}
\end{align*}
$$

The consistency of these equations implies that $x^{\alpha \beta}$ and $\theta_{a}^{\alpha}$ must transform according to eqs. (3.7) and (3.8), respectively, if the $y_{A}$ coordinates are transformed as in eq. (2.3). This indicates that we may identify these quantities with the usual coordinates in $(2,0)$ superspace.

Let us define a new constraint, similar to eq. (4.1), but this time based on the product in eq. (3.1). The most natural such condition is

$$
\begin{equation*}
\hat{I}^{A B} y_{A} y_{B}=0 . \tag{4.3}
\end{equation*}
$$

If we combine this condition with the supercone constraint (4.1), we find two separate equations:

$$
\begin{align*}
& 2 y^{\dot{\alpha}} y_{\dot{\alpha}}+\epsilon_{\dot{a} b} y^{\dot{a}} y^{\dot{b}}=0  \tag{4.4}\\
& 2 y^{\dot{\alpha}} y_{\dot{\alpha}}+\epsilon_{\dot{a} \dot{b}} y^{\dot{a}} y^{\dot{b}}=0 .
\end{align*}
$$

In analogy with eq. (4.2), we write an explicit solution to these equations as

$$
\begin{align*}
y^{\dot{a}} & =\sqrt{2} i \epsilon^{\dot{a} \dot{b}} \theta_{\dot{b}}^{\dot{\beta}} y_{\dot{\beta}} \\
y^{\dot{\alpha}} & =\left(2 x^{\dot{\alpha} \dot{\beta}}+\epsilon^{\dot{a} \dot{b}} \theta_{\dot{\alpha}}^{\dot{\alpha}} \theta_{\dot{b}}^{\dot{\beta}}\right) y_{\dot{\beta}}, \tag{4.5}
\end{align*}
$$

together with similar equations with primed indices. We see immediately that this defines a surface with two bosonic dimensions, which also is described by eight fermionic variables. This agrees with the results found in the preceding section and suggests that the surface in $(2,0)$ superspace may be regarded as the intersection of the supercone (4.1) and the hyper-surface (4.3) in the superconformal space.

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## References

[1] E. Witten, Some comments on string dynamics, hep-th/9507121.
[2] E.P. Verlinde, Global aspects of electric - magnetic duality, Nucl. Phys. B 455 (1995) 211 hep-th/9506011.
[3] M.B. Green, N.D. Lambert, G. Papadopoulos and P.K. Townsend, Dyonic p-branes from self-dual (p+1)-branes, Phys. Lett. B 384 (1996) 86 hep-th/9605146.
[4] M. Henningson, A class of six-dimensional conformal field theories, Phys. Rev. Lett. 85 (2000) 5280 hep-th/0006231.
[5] E. Witten, Conformal field theory in four and six dimensions. Prepared for Symposium on Topology, Geometry and Quantum Field Theory (Segalfest), Oxford, England, United Kingdom, 24-29 Jun 2002
[6] C. Montonen and D.I. Olive, Magnetic monopoles as gauge particles?, Phys. Lett. B 72 (1977) 117 .
[7] P. Goddard, J. Nuyts and D.I. Olive, Gauge theories and magnetic charge, Nucl. Phys. B 125 (1977) 1.
[8] H. Osborn, Topological charges for $N=4$ supersymmetric gauge theories and monopoles of spin 1, Phys. Lett. B 83 (1979) 321.
[9] O.J. Ganor, Six-dimensional tensionless strings in the large- $N$ limit, Nucl. Phys. B 489 (1997) 95 hep-th/9605201.
[10] J.M. Maldacena, Wilson loops in large-N field theories, Phys. Rev. Lett. 80 (1998) 4859 hep-th/9803002.
[11] D. Berenstein, R. Corrado, W. Fischler and J.M. Maldacena, The operator product expansion for wilson loops and surfaces in the large-N limit, Phys. Rev. D 59 (1999) 105023 hep-th/9809188.
[12] R. Corrado, B. Florea and R. McNees, Correlation functions of operators and wilson surfaces in the $D=6$, (0,2) theory in the large-N limit, Phys. Rev. D 60 (1999) 085011 hep-th/9902153.
[13] M. Henningson, Commutation relations for surface operators in six- dimensional (2,0) theory, JHEP 03 (2001) 011 hep-th/0012070.
[14] M. Henningson, Wilson-'t hooft operators and the theta angle, JHEP 05 (2006) 065 hep-th/0603188.
[15] J.M. Maldacena, The large- $N$ limit of superconformal field theories and supergravity, Adv. Theor. Math. Phys. 2 (1998) 231 hep-th/9711200.
[16] N. Drukker, D.J. Gross and H. Ooguri, Wilson loops and minimal surfaces, Phys. Rev. D 60 (1999) 125006 hep-th/9904191.
[17] N. Drukker and D.J. Gross, An exact prediction of $N=4$ susym theory for string theory, $J$. Math. Phys. 42 (2001) 2896 hep-th/0010274.
[18] M. Bianchi, M.B. Green and S. Kovacs, Instanton corrections to circular wilson loops in $N=4$ supersymmetric Yang-Mills, JHEP 04 (2002) 040 hep-th/0202003.
[19] P. Arvidsson, Manifest superconformal covariance in six-dimensional (2,0) theory, JHEP $\mathbf{0 3}$ (2006) 076 hep-th/0602193.
[20] P. Arvidsson, Superconformal theories in six dimensions, hep-th/0608014.
[21] V.G. Kac, A sketch of lie superalgebra theory, Commun. Math. Phys. 53 (1977) 31.
[22] W. Nahm, Supersymmetries and their representations, Nucl. Phys. B 135 (1978) 149.
[23] P. Arvidsson, Superconformal symmetry in the interacting theory of (2,0) tensor multiplets and self-dual strings, J. Math. Phys. 47 (2006) 042301 hep-th/0505197.
[24] P. Claus, R. Kallosh and A. Van Proeyen, M 5-brane and superconformal (0,2) tensor multiplet in 6 dimensions, Nucl. Phys. B 518 (1998) 117 hep-th/9711161.
[25] P. Arvidsson, E. Flink and M. Henningson, Free tensor multiplets and strings in spontaneously broken six-dimensional (2,0) theory, JHEP 06 (2003) 039 hep-th/0306145.
[26] J.-H. Park, Superconformal symmetry in six-dimensions and its reduction to four, Nucl. Phys. B 539 (1999) 599 hep-th/9807186;

